A <u>worked example</u>

of computing an integral using the definition.

$$\int_2^4 3x^3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where } f(x)=3x^3,\ \Delta x = \frac{4-2}{n} = \frac{2}{n}$$

$$\text{and } x_i = 2 + \frac{2i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} 3\left(2+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6}{n}\right)\left(8 + \frac{24i}{n} + \frac{24i^2}{n^2} + \frac{8i^3}{n^3}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{48}{n} + \frac{144i}{n^2} + \frac{144i^2}{n^3} + \frac{48i^3}{n^4}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \frac{48}{n} + \sum_{i=1}^{n} \frac{144}{n^2} i + \sum_{i=1}^{n} \frac{144i^2}{n^3} + \sum_{i=1}^{n} \frac{48i^3}{n^4}\right)$$

not enough room :-(

$$= \cancel{\lim_{n\to\infty} \left(\frac{48}{n}\cdot n + \frac{144}{n^2}\,\frac{n(n+1)}{2} + \frac{144}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{48}{n^4}\right.}$$

$$= \lim_{n\to\infty} \left(\frac{48}{n}\cdot n + \frac{144}{n^2}\,\frac{n(n+1)}{2} + \frac{144}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{48}{n^4}\left[\frac{n(n+1)}{2}\right]^2\right)$$

$$= \lim_{n\to\infty} \left(48 + \frac{72(n(n+1))}{n^2} + \frac{24}{n^3}\,n(n+1)(2n+1) + \frac{12}{n^4}(n(n+1))^2\right)$$

$$= 48 + 72 + 48 + 12$$

$$= 180$$